



Fault size distributions — are they really power-law?

A. NICOL, J. J. WALSH, J. WATTERSON and P. A. GILLESPIE

Fault Analysis Group, Department of Earth Sciences, University of Liverpool, Liverpool L69 3BX, U.K.

(Received 17 November 1994; accepted in revised form 5 July 1995)

Abstract—Power-law fault size (throw) distributions spanning ca seven orders of magnitude are analysed using sample lines from seismic, coal-mine plan and outcrop data sets. Outcrop data sets generally have lower power-law exponent values than seismic data sets, consistent with a general decrease in exponent value with decrease in fault size. If such a relationship does exist, then probably it cannot be accounted for by sampling bias alone. We suggest that either: (i) fault size populations are not power-law over a large scale range; or (ii) they are power-law, but at smaller scales show a greater range of exponent values due to spatial clustering.

INTRODUCTION

In recent years, many authors, including ourselves, have claimed that the size populations of faults are often described by power-law distributions (Kakimi 1980, Childs *et al.* 1990, Scholz & Cowie, 1990, Marrett & Walsh *et al.* 1991, Allmendinger 1992, Jackson & Sanderson 1992, Yielding *et al.* 1992), where size is expressed either as displacement or dimension. This relationship can be expressed as

$$N \propto S^{-C}$$

where N is the number of faults with size greater than or equal to S and C is the power-law exponent. When this function is plotted on log-log axes, a straight line of slope $-C$ results. Fault systems analysed using either line (one-dimensional) or map (two-dimensional) samples have a range of C values; line sample throw populations have, for example, C values in the range 0.4–1.0. The power-law nature of fault size distributions are consistent with fault systems being either self-similar or self-affine fractals and provides a basis for both quantitative definition of a fault size scaling law and predictions beyond data windows. For some time, we have been concerned that there appears to be a systematic variation of the results according to the type of fault data used. The C value derived from outcrop data often seemed to have values towards the lower end of the range (ca 0.4–0.6) while values from seismic data sets tended towards the higher end (ca 0.8–1.0). Values derived from coal-mine data appeared to be concentrated towards the middle of the range (ca 0.5–0.8). Although size distributions of larger faults tend to have higher exponent values than do smaller faults, the existence of a systematic relationship is still uncertain. Such a relationship could arise from one or more of the following possibilities.

(i) Fault population size distribution curves are not straight when a wide enough range of fault sizes ($>$ ca two orders of magnitude) is included in the sample. If the size distribution of faults over the full size range is described by a relatively gentle curve of some type,

individual parts of the curve spanning only one–two orders of magnitude could well appear to be straight.

(ii) Data from outcrops, coal-mine plans and seismics differ not only in the sizes of faults which are sampled, but also in their sampling biases. For example, a sampling bias favouring the sampling of lower C value subsets at smaller scales (i.e. outcrop data sets) could occur if the spatial distribution of faults within a fault system is such that C varies spatially (Yielding *et al.* 1992, Walsh *et al.* 1994).

(iii) That smaller faults form distinct populations within individual lithological units, whereas larger faults form a single population controlled by the ‘average’ lithology of the sequence which they intersect (Walsh *et al.* 1994).

The difficulty of deciding between these three possibilities is almost entirely due to the restricted ranges of individual data sets. Whereas the size range of faults in a single fault system may span up to eight orders of magnitude, the size range in an individual data set is rarely greater than two orders of magnitude. In only a few cases (Walsh *et al.* 1991, Yielding *et al.* 1992) has the power-law size distribution of faults been tested on single data sets spanning more than three orders of magnitude of fault size.

THE DATA

The fault size distributions used are those obtained by one-dimensional sampling of throw values from normal fault systems (Childs *et al.* 1990, Walsh *et al.* 1994). A previous attempt to test for a systematic relationship between C and data type in one-dimensional samples appeared to show that none exists (Walsh *et al.* 1994). We have now repeated the test, but have excluded from consideration all data sets which do not meet three quality criteria when plotted as log cumulative number vs log throw size. A sample must include 30 or more fault throw values (for single-line data sets used, the average number is 152 and the average number is 432 for multi-

line data sets). The central part of the distribution must define a straight line; stepped lines resulting from rounding of throw measurements are accepted. The straight-line segment of each plot must extend over a minimum range of one order of magnitude of throw size. The sources and characteristics of the 33 data sets meeting these criteria are shown in Table 1. Although 53 data sets did not meet the criteria, the criteria are nevertheless insufficiently rigorous. Three data sets which stand out as significantly different from the others on most plots would probably have been excluded had more rigorous selection criteria been adopted; these are the Flamborough Head, Cumbrian Opencast and Star Crossing data sets. The Flamborough Head data set is the only one which includes throw measurements from a significant number (ca 24%) of strike-slip and oblique-slip faults (Peacock & Sanderson 1994) and should probably have been excluded. The Cumbrian Opencast data set derives from an exploration coal-seam plan constructed from widely spaced (ca 40 m) borehole data and is believed to be of low quality and, with hindsight, should have been rejected. The Star Crossing data are from faulted glacial sands which marks them as atypical, but two other data sets from glacial sands which are included do not stand out from the other data sets on any plot.

Data from the selected 33 data sets are plotted in Fig. 1 as a slope vs mid-point throw, with individual data sets categorized by the type of source data, i.e. outcrop, coal-mine plans, seismic. The three distinctive data sets are also identified. The mid-point throw is the throw at the mid-point of the straight-line segment of the size distribution curve and slope is the absolute value of the gradient of the straight-line segment, i.e. C . There is a tendency for slope values to decrease with decrease in mid-point size; this decrease in slope is especially marked if the three distinctive data sets are excluded from consideration. Slope values for outcrop data are generally less than those for seismic data sets, while coal-mine data occupy the middle of the range. Given the relatively small number of data sets and the relatively wide variation in slope at any given mid-point size, the relationship between slope and mid-point range could be fortuitous. We take the view that the correlation is probably real but is certainly not proven. If real, the question is whether or not this relationship represents a genuine change in scaling properties of faults with change in size, or whether it is due to some other cause, e.g. sampling bias.

SAMPLING BIAS

Given the wide size range of faults which occur, the use of varied sampling techniques cannot be avoided and there is no certainty whether any technique provides a size sample which is 'representative' of the whole. Sample attributes which could possibly influence the size distribution systematics include variations in fault density, sample-line length and sample size range.

Fault density

Fault densities vary with the type of data set for both cultural and logistical reasons. Outcrops which are sampled tend to be those with moderate-high densities because of the requirement for a relatively large number of faults along an uninterrupted outcrop. The highest fault densities are unlikely to be sampled in outcrop because intensely fractured outcrops are usually poorly exposed and high fracture densities are uncommon in temporary exposures, opencast coal workings for example, because densely faulted sequences are uneconomic to work. Underground coal-mine data sets tend to have low densities because even moderately faulted areas cannot be mined economically.

Some oilfield data sets may also have low fault densities but, given typical resolutions and survey areas, low density data sets will usually have too few faults to provide an acceptable sample. Highly faulted reservoirs, on the other hand, may give rise to poor quality seismic data or may even be uneconomic and not seismically surveyed. The tendency for moderate fault densities to be over-represented in coal-mine and seismic data sets is reinforced by the common use of multi-line sampling which enables a robust data set to be obtained with fewer throw readings per km sample line than is possible with single-line sampling. All but two of the coal-mine and seismic data sets used here are multi-line samples with between 10 and 60 lines. Use of a multi-line sample appears not to influence the C value (Childs *et al.* 1990).

The possibility has been investigated that systematic density differences between the different data set types could be responsible for the positive correlation between mid-point size (which is dependent on the type of data) and C (Fig. 1) by plotting C vs standardized fault density (Fig. 2). Standardized fault density is taken as the density of fault throws above a specified size, which is the same for all the data sets and is here taken as 1 m. Determination of the standardized density of a seismic data set requires downwards extrapolation to 1 m of the straight-line segment of the size distribution curve. The curves for some of the outcrop data sets require extrapolation up to 1 m throw. Density estimates obtained using these limited extrapolations are not significantly affected by whether or not the curves are strictly power-law.

Although standardized density values for outcrop data are often higher than those for seismic and coal-mine data, Fig. 2 shows no clear relationship between standardized density and C . The effect on C of density variations within individual data sets or fault systems is considered later.

Sample-line length

Sampled fault size distributions could vary systematically with sample-line lengths. A correlation between C and sample-line length would be inevitable if C varied systematically with data type, given that sample-line length varies systematically with data type, i.e. seismic

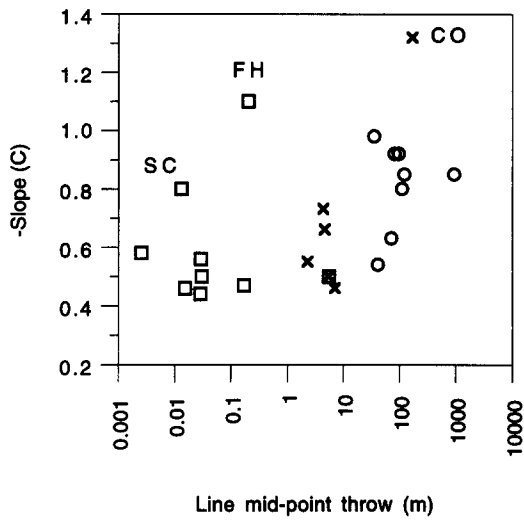


Fig. 1. Exponent of the size distribution, C , vs line mid-point throw for 22 outcrop (open squares), coal-mine plan (crosses) and off-shore seismic (open circles) data sets from the U.K. and North Sea (see Table 1). Note that where multiple populations are sampled from the same locality, average values weighted for the number of faults in each sample are plotted. Line mid-point throw is the throw at the mid-point of the straight-line segment of the one-dimensional throw population curve and C is determined by fitting a line by eye to the straight-line segment of each data set and is equivalent to negative slope of the segment. Symbols: SC = Star Crossing, FH = Flamborough Head, CO = Cumbrian Opencast (see Table 1 for further details).

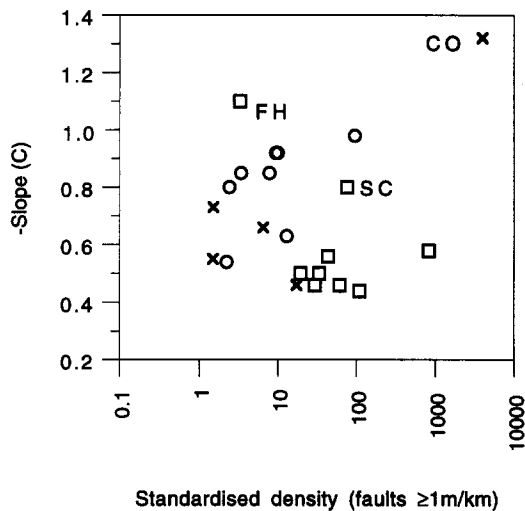


Fig. 2. Exponent of the size distribution, C , vs fault density for each of the locations in Table 1. Standardized fault densities are derived for faults with throws ≥ 1 m/km of sample line. Calculation of standardized densities required extrapolation of both seismic and of some outcrop power-law curves to CUMFD (cumulative number of faults per km of sample-line length) at 1 m fault throw and are therefore not wholly independent of the estimated C ; despite this factor, no clear relationship is observed between density and slope. Symbols and notation as for Fig. 1.

and mine-plan sample lines are nearly always longer than outcrop sample lines. However, C shows no correlation with sample-line length (Fig. 3a). A plot (Fig. 3b) of slope vs number of faults on single-line samples shows a weak correlation for all but three data sets; this is not surprising, since the number of faults in a given size range will vary with C .

The effect on C of the ratio of sample-line length to fault size can be examined on a plot of maximum throw

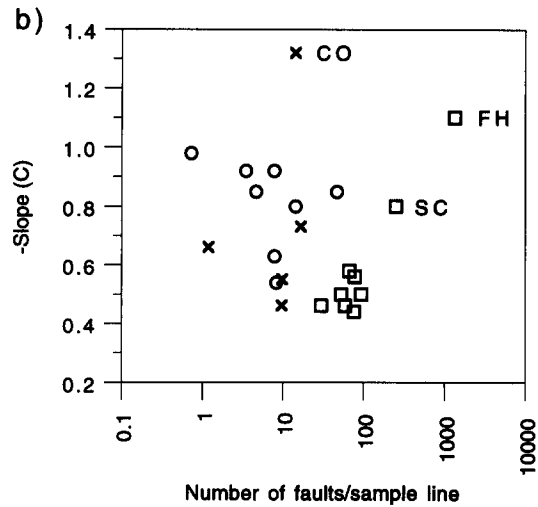
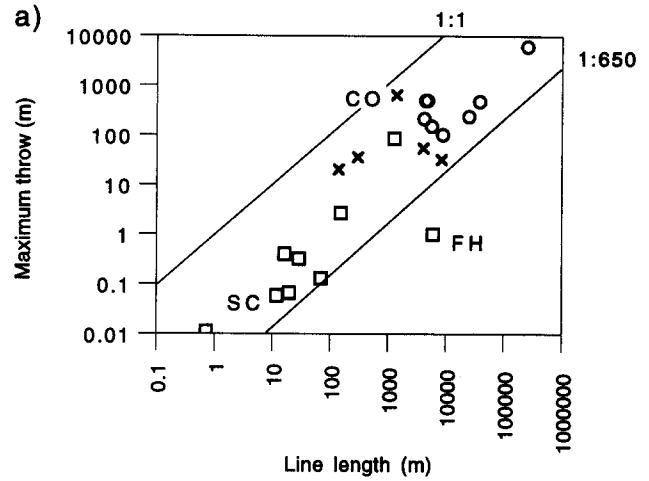


Fig. 3. (a) Maximum throw vs sample-line length; for multi-line samples average maximum throws and line lengths are used. Maximum throw is the throw at the right-hand end of the straight-line segment of the size distribution curve. (b) Exponent of the size distribution, C , vs number of faults per sample line. Symbols and notation as for Fig. 1.

sampled vs sample-line length (Fig. 3a). The data define a broad band of points with a slope of ca 1.0 and upper and lower boundaries with sample-line length/maximum throw ratios of 1 and 650, respectively. This relationship is to be expected as larger faults are more likely to be encountered on longer sample lines. Most data sets on Fig. 3(a) lie in the centre or lower part of the band with ratios in the range 20–100 but do not vary sufficiently to give rise to a systematic variation of C .

Sample-size range

The crucial importance of the throw size ranges of samples has been demonstrated previously (Childs *et al.* 1990, Gillespie *et al.* 1993, Walsh *et al.* 1994) and a minimum valid size range of one order of magnitude has been suggested. All data sets suffer to some extent from scale ranges limited by throw truncation at the lower end and incomplete sampling of larger throws at the other end (Pickering *et al.* 1994). Truncation can result in a decrease in C at the small end of the range, whereas censoring effects may result in increased C values towards the higher end. For seismic data, because many

seismic surveys of reservoirs are bounded by major faults, which are excluded from the data area, large throws may either be under-sampled or not sampled at all (Childs *et al.* 1990). Large faults may be excluded from outcrop samples because they are preferentially eroded and therefore tend to terminate sample lines. As these factors can modify and degrade both the upper and lower ends of straight-line segments of population curves, they reduce the effective scale range of the data. Table 1 shows that size ranges for individual outcrop data sets can be up to three–four orders of magnitude, e.g. 4 mm–11 m, while the size ranges of coal-mine and seismic data sets are generally less than two orders of magnitude. For our selected data sets, with well-defined truncation values, the limited scale ranges could explain some of the steepening of population slopes for coal-mine and seismic data sets.

Large scale range populations from outcrop are unlikely to give high C values, since a value close to 1.0 for a size range > two orders of magnitude would require more than 100 faults in the sample. So many faults require either a very long sample line or a very high fault density, neither of which is usual for either natural outcrop or quarry samples. Furthermore, good population curves with low C values require large scale range data sets, so low C values are less likely to be obtained from seismic data than from outcrop data because of the typically greater scale ranges of the latter. However, when outcrop and seismic samples of the same scale range are compared, slopes from seismic data are still generally higher than those from outcrop data. There are, however, more fundamental reasons which might be responsible for, or contribute to, the observed relationship between slope and fault size.

GEOLOGICAL EFFECTS

Spatial distribution of faults

Some types of spatial distribution of faults could lead to systematic differences of size distribution between samples on different scales (see Peacock & Sanderson 1994), so that a sample is 'representative' of the fault system only on its own particular scale. Although techniques have been proposed for analysing spatial distributions of faults in one-dimensional samples (e.g. Velde *et al.* 1990) they are of doubtful validity and fault clustering has not been satisfactorily quantified (Gillespie *et al.* 1993). It is therefore not known whether or not clustering is scale dependent. In a clustered fault system, the size distributions sampled either by line sampling (one-dimensional) or by map sampling (two-dimensional) may vary with sample position relative to clusters. In these circumstances, sub-sets of line samples may each provide populations with different characteristics, each of which is 'unrepresentative' either of the whole sample or of the fault system as a whole. Without a quantitative description of the systematics of fault spatial distributions, it is not possible to determine

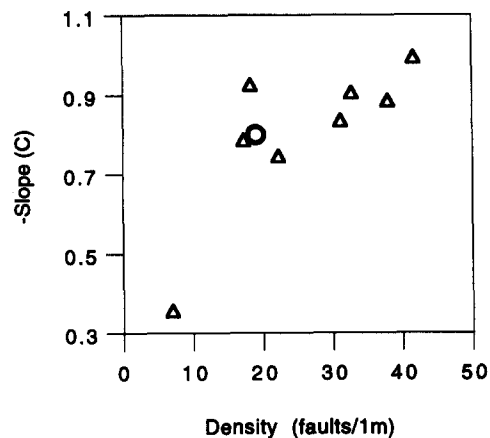


Fig. 4. Exponent of the size distribution, C , vs fault density for sub-samples of the Star Crossing data set. This data set contained 251 faults measured over a total sample-line length of 12 m and was sub-sampled using a 4 m wide window moved in 1 m increments. Window centres were positioned at distances of 2–10 m along the sample line to produce nine sub-samples, eight of which provided sufficient data (>25 faults) and are plotted as open triangles. Densities represent the number of faults observed per metre of sample-line length. The average number of faults in each sub-sample is 94. Open circle shows the density and slope for the entire sample.

whether or not population characteristics, including power-law slopes, vary with scale.

The combined effects of fault density and sample-line length have been considered above in terms of data sets from a variety of sources, but these effects can also be investigated by sub-sampling large single data sets. The Star Crossing data set, with throw data for 251 faults, has been sub-sampled using 4 m windows moved in 1 m increments along the 12 m sample-line, and Fig. 4 shows plots of C and of density for the sub-samples. Sub-samples which provided acceptable curves with > 25 faults show a broad correlation between density and C , with the highest C value recorded for the highest density sample window. A similar result is given by the map data in Yielding *et al.* (1992) which show that both the density and population characteristics (including value of C) of each sub-area differ from those of the whole map area. This density variation reflects the heterogeneous nature of fault arrays when sampled at smaller scales, e.g. outcrop, although the population characteristics of a larger scale sample, e.g. reservoir seismic, may well be 'representative' of a larger area. The increase in variability with decrease in scale is compatible with population curves projected to smaller scales representing only an average of the many different small scale populations (Fig. 5). By analogy with Star Crossing and with the map data of Yielding *et al.* (1992), it is feasible that sub-areas of low fault density not only have shallower power-law slopes but represent a relatively large proportion of the whole sample area. Areas of higher fault density, on the other hand, may be characterized by steep population curves and will therefore have exaggerated effects on population curves for entire populations.

The variability of densities and other fault population attributes at smaller scales is illustrated in Fig. 5 for fault throw populations from a North Sea oilfield, derived

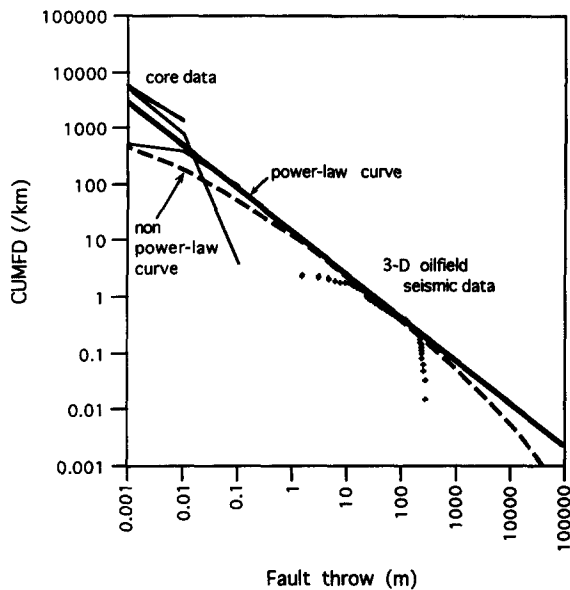


Fig. 5. Empirical non-power-law curve (see text for details) and power-law curve (labelled), showing their relationships to displacement population curves for faults mapped from three-dimensional seismic data and in core (labelled), for the same oilfield (data from Walsh *et al.* 1991).

from seismic data and from cores. The range of core derived fault densities extends down to the density of the non power-law curve but straddles that of the extrapolated power-law seismic data curve. The lowest density population curve also has the lowest C value which is consistent with sympathetic variation of C and of density at small scales.

Lithological control

With the exception of the Flamborough Head data set, the data sets used are for faults which have large vertical extents relative to lithological variations on the scale of individual beds; the results obtained are therefore not affected by this scale of lithological variation.

At larger scales, the population characteristics of faults may be influenced by the mechanical properties of a faulted sequence (Peacock & Sanderson 1994) and the thickness systematics of sedimentary sequences could influence the scaling of within-sequence faults by introducing a characteristic length scale (e.g. Gillespie *et al.* 1993, fig. 2e). The absence of a characteristic length scale is crucial to formation of power-law, or fractal, populations. In an individual data set, sequence effects are likely to be more significant at smaller scales, and some variation of C might be expected in populations of small faults derived from outcrop or core (Walsh *et al.* 1994). Less competent layers will accommodate a relatively higher proportion of strain by plastic processes than by faulting. More competent units may contain more faults, the sizes and spatial distributions of which may be controlled by layer thickness. Fault densities and population characteristics measured in outcrop or core could, therefore, be systematically different from those predicted by extrapolation from seismic data. Our data do not, however, provide evidence to suggest that the

relationship between C and mid-point throw (Fig. 1) could be accounted for by lithological factors.

Temporal changes in C

On the bases of numerical modelling (Cowie *et al.* 1993, 1994) and outcrop data (Wojtal 1994, 1996), it has been suggested that C decreases as a fault system evolves due to progressive concentration of strain onto large faults. A relationship of this type is difficult to reconcile with maintenance of a power-law earthquake population throughout the life of a fault system (Walsh & Watterson 1992). If the strain accommodated by a fault system is taken as a measure of maturity, then the higher strains associated with our off-shore North Sea data sets, compared with the onshore outcrop and coal-mine data sets, indicate that the off-shore fault systems are more mature than the on-shore fault systems which we have sampled. As the C values for the off-shore systems are generally higher than those for the on-shore faults, our data are inconsistent with the C values having a largely temporal control, although our data do not exclude the possibility of a C value which decreases with growth of an individual fault system.

DISCUSSION AND CONCLUSIONS

We have examined possible sampling biases due to spatial variations in fault density, sample-line length, data resolution, sequence lithology and differences in sampling methodology between, or within, different data types, i.e. outcrop, coal-seam plan and seismic interpretation. It is concluded that the general decrease in C value with decreasing fault size cannot be ascribed to sampling bias alone. Data sets which become available in the future may show that C -values for outcrop samples can range up to those typical for seismic data sets more often than our data indicate.

The apparent relationship between fault size and C could indicate that fault size populations are not power-law but conform to a fault scaling law represented by the gentle convex-upwards curve shown in Fig. 5 which is consistent with the relationship between C and fault size indicated by Fig. 1. The non-power-law empirical curve is derived by least-squares regression of the data in Fig. 1, from which $C = 0.07 \log(t) + 0.68$ ($C = -\text{slope}$ and $t = \text{mid-point throw}$), and pinning the curve to the mid-point throw of the seismic data. The curvature of the empirical curve is so slight that, over two orders of magnitude of fault size, it is practically indistinguishable from a straight line. For most practical purposes, the distinction between the curve and a straight line is unimportant so long as extrapolations are restricted to ca two orders of magnitude. A non-power-law relationship would indicate that either different processes or different controls are effective on different scales. Similarly, the concentrations of small faults immediately adjacent to larger faults, sometimes collectively referred

to as damage zones, may provide distinct size populations (e.g. Knipe *et al.* 1994).

An additional realistic possibility is that fault size populations are power-law, but show greater ranges of C values and densities at smaller scales due to the spatial systematics of the fault distribution. Clustered fault systems provide increasingly heterogeneous fault densities at smaller scales (Gillespie *et al.* 1993). Sympathetic variation of C with fault density would, for a given fault system, provide a higher proportion of low C value populations at outcrop scale than at a larger sampling scale. Although a fault system may show a range of fault densities and slopes for different outcrop scale samples, a larger scale power-law population curve would still be 'representative' of the system as a whole. The heterogeneity of fault densities and populations at smaller scales is illustrated by the range of core-derived fault densities shown in Fig. 5.

The available data suggest that size population slopes of outcrop scale samples are usually shallower than those for larger scale samples. This difference may reflect either a non-power-law scaling law for fault size distribution, or the spatial systematics of fault arrays at smaller scales.

Acknowledgements—This work was carried out for projects funded by an OSO/NERC Hydrocarbon Reservoirs Link Programme project (No. 7137) on Faulted Reservoir Analysis System and by the E.U. JOULE II Hydrocarbons Programme Reservoir Engineering project (contract No. J0U2-CT92-0182). We are grateful to colleagues in several companies for their assistance in providing access to data and thank Isabel Jones for drafting Table 1 and Fig. 5. Thanks also to Randy Marrett, Richard Allmendinger and Steven Wojtal for their constructive reviews.

REFERENCES

- Childs, C., Walsh, J. J. & Watterson, J. 1990. A method for estimation of the density of fault displacements below the limits of seismic resolution in reservoir formations. In: *North Sea Oil and Gas Reservoirs II* (edited by Buller, A. T., Berg, E., Hjelmeland, O., Kleppe, J., Torsæter, O. & Aasen, J. O.). Graham & Trotman, London, 309–318.
- Cowie, P. A., Vanneste, C. & Sornette, D. 1993. Statistical physics model for the spatio-temporal evolution of faults. *J. geophys. Res.* **98**, 21,809–21,821.
- Cowie, P. A., Vanneste, C. & Sornette, D. 1994. Multifractal scaling properties of a growing fault population. Tectonic Studies Group Fault Populations Meeting (19–20 October 1994). *Ext. Abst.* 42–43.
- Gillespie, P. A., Howard, C. B., Walsh, J. J. & Watterson, J. 1993. Measurement and characterisation of spatial distributions of fractures. *Tectonophysics* **226**, 113–141.
- Jackson, P. & Sanderson, D. J. 1992. Scaling of fault displacements from the Badajoz–Córdoba shear zone, SW Spain. *Tectonophysics* **210**, 179–190.
- Kakimi, T. 1980. Magnitude–frequency relation for displacement of minor faults and its significance in crustal deformation. *Bull. geol. Surv. Japan* **31**, 467–487.
- Knight, J. L. 1990. Displacement geometry of normal faults. British Coal Opencast Executive, unpublished report (No. 1330F).
- Knight, J. L. 1992. Fault analysis of the Buckhead–Esperly Lane opencast site, south-west Durham. British Coal Opencast Executive, unpublished report (No. 6056F).
- Knipe, R. J., Baxter, K., Clennell, M. B., Farmer, A. B., Fisher, Q. J., Jones, G., Bolton, A. J., Duerden, M., Kidd, B. E., Porter, J. R. & White, E. A. 1994. Fault populations: fault array evolution processes and populations. Tectonic Studies Group Fault Populations Meeting (19–20 October 1994). *Ext. Abstr.* 37–38.
- Marrett, R. & Allmendinger, R. W. 1992. Amount of extension on 'small' faults: an example from the Viking graben. *Geology* **20**, 47–50.
- Peacock, D. C. P. & Sanderson, D. J. 1994. Strain and scaling of faults in the chalk at Flamborough Head, U.K. *J. Struct. Geol.* **16**, 97–107.
- Pickering, G., Bull, J. M., Sanderson, D. J. & Harrison, P. V. 1994. Fractal fault displacements: a case study from the Moray Firth, Scotland. In: *Fractals and Dynamic Systems in Geosciences* (edited by Kruhl, J. H.). Springer-Verlag, Berlin.
- Scholz, C. H. & Cowie, P. A. 1990. Determination of total strain from faulting using slip measurements. *Nature* **346**, 837–839.
- Velde, B., Dubois, J., Touchard, G. & Badri, A. 1990. Fractal analysis of fractures in rocks: the Cantor's Dust method. *Tectonophysics* **179**, 345–352.
- Walsh, J. J. & Watterson, J. 1992. Populations of faults and fault displacements and their effects on estimates of fault-related regional extension. *J. Struct. Geol.* **14**, 70–712.
- Walsh, J. J., Watterson, J. & Yielding, G. 1991. The importance of small-scale faulting in regional extension. *Nature* **351**, 391–393.
- Walsh, J. J., Watterson, J. & Yielding, G. 1994. Determination and interpretation of fault size populations: procedures and problems. In: *North Sea Oil and Gas Reservoirs III* (edited by Aasen, J. O., Berg, E., Buller, A. T., Hjelmeland, O., Holt, R. M., Kleppe, J. & Torsæter, O.). Graham & Trotman, London, 141–145.
- Wojtal, S. F. 1994. Fault scaling laws and the temporal evolution of fault systems. *J. Struct. Geol.* **16**, 603–612.
- Wojtal, S. F. 1996. Changes in fault displacement populations correlated to linkage between faults. *J. Struct. Geol.* **18**, 265–279.
- Yielding, G., Walsh, J. J. & Watterson, J. 1992. The prediction of small-scale faulting in reservoirs. *First Break* **10**, 449–460.